

# Nonperturbative vacuum and condensates in QCD below thermal phase transition

N.O.Agasian\*

Institute of Theoretical and Experimental Physics,  
Moscow 117218, Russia

## Abstract

Thermodynamic properties of the QCD nonperturbative vacuum with two light quarks are studied. It is shown that at low temperatures  $T \lesssim M_\pi$  relativistic massive pions can be treated within the dilute gas approximation. Analytic temperature dependence of the quark condensate is found in perfect agreement with the numerical calculations obtained at the three-loop level of the chiral perturbation theory with non-zero quark mass. The gluon condensate slightly varies with the increase of the temperature. It is shown that the temperature derivatives of the anomalous and normal (quark massive term) contributions to the trace of the energy-momentum tensor in QCD are equal to each other in the low temperature region.

PACS:11.10.Wx,12.38.Aw,12.38.Mh

---

\*E-mail: [agasian@heron.itep.ru](mailto:agasian@heron.itep.ru)

1. The investigation of the vacuum state behavior under the influence of various external factors is known to be one of the central problems of quantum field theory. In the realm of strong interactions (QCD) the main factors are the temperature and the baryon density. At low temperatures,  $T < T_c$  ( $T_c$ —temperature of the "hadron–quark–gluon" phase transition), the dynamics of QCD is essentially nonperturbative and is characterized by confinement and spontaneous breaking of chiral symmetry (SBCS). In the hadronic phase the partition function of the system is dominated by the contribution of the lightest particles in the physical spectrum. In QCD this role is played by the  $\pi$ -meson which is the Goldstone excitation mode in chiral condensate in the limit of two massless flavors. Therefore the low temperature physics can be described in terms of the effective chiral theory [1, 2, 3]. An important problem is the behavior of the order parameter (the quark condensate  $\langle \bar{q}q \rangle$ ) with the increase of the temperature. In the ideal gas approximation the contribution of the massless pions into the quark condensate is proportional to  $T^2$  [4, 5]. In chiral perturbation theory (ChPT) the two – and three–loop contributions ( $\sim T^4$  and  $\sim T^6$  correspondingly) into  $\langle \bar{q}q \rangle$  have been found in [5] and [6, 7]. In the general case of  $N_f$  massless flavors, the low temperature expansion of the quark condensate takes the form

$$\frac{\langle \bar{q}q \rangle(T)}{\langle \bar{q}q \rangle} = 1 - \frac{(N_f^2 - 1)}{N_f} \frac{T^2}{12F_\pi^2} - \frac{(N_f^2 - 1)}{2N_f^2} \left( \frac{T^2}{12F_\pi^2} \right)^2 - N_f(N_f^2 - 1) \left( \frac{T^2}{12F_\pi^2} \right)^3 \ln \frac{\Lambda_q}{T} + O(T^8), \quad (1)$$

where  $F_\pi \simeq 93\text{MeV}$  is the value of the pion decay constant and  $\Lambda_q \simeq 470\text{MeV}$  ( $N_f = 2$ ) [7]. The result (1) is an exact theorem of QCD in chiral limit.

The situation with the gluon condensate  $\langle G^2 \rangle \equiv \langle (gG_{\mu\nu}^a)^2 \rangle$  is very different. The gluon condensate is not an order parameter of the phase transition and it does not lead to any spontaneous symmetry breaking (SSB). At the quantum level the trace anomaly leads to the breaking of the scale invariance which is phenomenologically described by non-zero value of  $\langle G^2 \rangle$ . However, this is not a SSB phenomenon and hence does not lead to the appearance of the Goldstone particle. The mass of the lowest excitation (dilaton) is directly connected to the gluon condensate,  $m_D \propto (\langle G^2 \rangle)^{1/4}$ . Thus, in the gluonic sector of QCD, the thermal excitations of glueballs are exponentially suppressed by the Boltzmann factor  $\sim \exp\{-m_{gl}/T\}$  and their contribution to the shift of the gluon condensate is small ( $\Delta\langle G^2 \rangle / \langle G^2 \rangle \sim 0.1\%$  at  $T \sim 200\text{ MeV}$ ) [8]. Next we note that in the one-loop approximation of ChPT pions are described as a gas of massless noninteracting particles. Such a system is obviously scale-invariant and therefore does not contribute to the trace of the energy-momentum tensor and correspondingly to  $\langle G^2 \rangle$ . As it has been demonstrated in [9] the gluon condensate temperature dependence arises only at the ChPT three-loop level due to the interaction among the Goldstone bosons.

For the quark condensate the account of the non-zero quark mass was done in [7]. Within ChPT two masses of the  $u$  and  $d$  quarks are considered as perturbation ( $m_u$  and  $m_d$  are much smaller than any scale of the theory). Correspondingly thermodynamic quantities (pressure,  $\langle \bar{q}q \rangle(T)$ ) are represented as a series in powers of  $T$  and  $m$  in low temperature expansion. As a consequence of the fact that the spectrum of the unperturbed system contains massless pions, the quark mass term generates infrared singularities involving negative powers of  $T$  [7]. In Ref. [7] results of a numerical procedure was presented in the form of the graphical dependence of  $\langle \bar{q}q \rangle(T)$  as a function of  $T$ , which we will compare with our analytic formula. The deviation of the quark condensate from the chiral limit (1) turns out to be substantial ( $\sim 25\%$ ) at  $T = 140\text{ MeV}$  in both cases.

It is well known that due to the smallness of pion mass as compared to the typical scale of strong interactions, the pion plays a special role among other strongly-interacting particles. Therefore for many problems of QCD at zero temperature the chiral limit,  $M_\pi \rightarrow 0$ , is an appropriate one. On the other hand a new mass scale emerges in the physics of QCD phase transitions, namely the critical transition temperature  $T_c$ . Numerically the critical transition temperature turns out to be

close to the pion mass,  $T_c \approx M_\pi$ .<sup>1</sup> Thus in the confining phase of QCD the small parameter  $M_\pi/T$  is lacking and hence the interval  $M_\pi \ll T < T_c$  where the high temperature expansion is applicable for thermal pions does not exist. However hadron states heavier than pion have masses several times larger than  $T_c$  and therefore their contribution to the thermodynamic quantities is damped by Boltzmann factor  $\sim \exp\{-M_{hadr}/T\}$ . Thus the thermodynamics of the low temperature hadron phase,  $T < M_\pi$ , is described basically in terms of the thermal excitations of relativistic massive pions.

In the present paper we study the behavior of the quark and gluon condensates in the low temperature phase. It is shown that the relativistic dilute massive pion gas approximation is applicable at temperature  $T \lesssim M_\pi$ . Analytical expressions for condensates are obtained. The comparison with the results of three-loop ChPT calculations  $\langle \bar{q}q \rangle(T)$  and  $\langle G^2 \rangle(T)$  is made. The low temperature relation for the trace of the energy-momentum tensor in QCD with two light quarks is obtained based on the general dimensional and renormalization-group properties of the QCD partition function and dominating role of the pion thermal excitations in the hadronic phase.

2. For non-zero quark mass ( $m_q \neq 0$ ) the scale invariance is broken already at the classical level. Therefore the pion thermal excitations would change, even in the ideal gas approximation, the value of the gluon condensate with increasing temperature. To determine this dependence can be use the general renormalization and scale properties of the QCD partition function. This is a standard method and it is used for derivation of low-energy QCD theorems [11]. For QCD at finite temperature and chemical potential these theorems were derived in [12, 13]. This method was used for investigation of QCD vacuum phase structure in a magnetic field [14] and at finite temperature [15] and also in nuclear matter [16].

A relation between the trace anomaly and thermodynamic pressure in pure-gluon QCD was obtained in [17] by making use of the dimensional regularization in the framework of the renormalization group (RG) method. Also within the RG method, but by employing slightly different techniques an analogous relation was derived in the theory with quarks in ref. [18]. Then, by making use of the results obtained in the above-mentioned papers, we can write the following expression for the trace anomaly in QCD with quarks:

$$\langle \theta_{\mu\mu}^g \rangle = \frac{\beta(\alpha_s)}{16\pi\alpha_s^2} \langle G^2 \rangle = -(4 - T \frac{\partial}{\partial T} - \sum_q (1 + \gamma_{m_q}) m_q \frac{\partial}{\partial m_q}) P_R, \quad (2)$$

where  $P_R$  is the renormalized pressure,  $\beta(\alpha_s) = d\alpha_s(M)/d \ln M$  is the Gell-Mann-Low function and  $\gamma_{m_q}$  is the anomalous dimension of the quark mass. It is convenient to choose such a large scale that one can take the lowest order expressions,  $\beta(\alpha_s) \rightarrow -b\alpha_s^2/2\pi$ , where  $b = (11N_c - 2N_f)/3$  and  $1 + \gamma_{m_q} \rightarrow 1$ . Thus, we have the following equations for condensates

$$\langle G^2 \rangle(T) = \frac{32\pi^2}{b} (4 - T \frac{\partial}{\partial T} - \sum_q m_q \frac{\partial}{\partial m_q}) P_R \equiv \hat{D} P_R, \quad (3)$$

$$\langle \bar{q}q \rangle(T) = - \frac{\partial P_R}{\partial m_q}. \quad (4)$$

3. The density of the pionic gas at temperature  $T$  is given by

$$n_\pi(T) = 3 \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp(\sqrt{\mathbf{p}^2 + M_\pi^2}/T) - 1}, \quad (5)$$

and the average distance between particles in this gas is  $L_\pi \simeq n_\pi^{-1/3}$ . The dilute gas approximation is valid, when the average distance  $L_\pi$  is much smaller than the mean free path  $\lambda_{\pi\pi}$  calculated

---

<sup>1</sup>The deconfining phase transition temperature is the one obtained in lattice calculations  $T_c(N_f = 2) \simeq 173$  MeV and  $T_c(N_f = 3) \simeq 154$  MeV [10].

taking into account collisions,  $L_\pi \ll \lambda_{\pi\pi}$ . For a hadronic gas, the mean free path is well known [19, 20] at low temperatures, where the gas almost exclusively consists of pions. An analysis of the  $\pi\pi$  collision rate shows that in the range  $50 \text{ MeV} < T < 140 \text{ MeV}$ , ChPT formula for mean free path  $\lambda_{\pi\pi} \simeq 12F_\pi^4/T^5$  [20] is valid to within about 20% [21]. In Fig.1 the behavior of the ratio  $L_\pi/\lambda_{\pi\pi}$  as a function of  $T$  is shown. Hence at  $T \lesssim M_\pi$  the gas approximation for pions can be used.

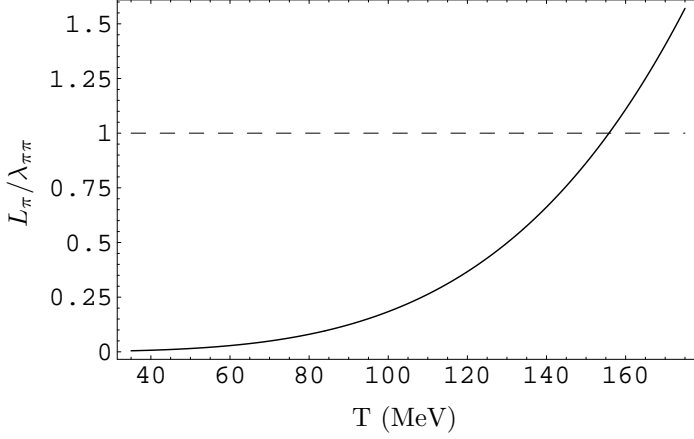


Figure 1: The ratio of the mean interparticle distance  $L_\pi$  ( in pionic gas ) to the mean free path  $\lambda_{\pi\pi}$  as a function of the temperature

Thus the effective pressure from which one can extract the condensates  $\langle \bar{q}q \rangle(T)$  and  $\langle G^2 \rangle(T)$  using the relations (3) and (4) has the form

$$P_{eff}(T) = -\varepsilon_{vac} + P_\pi(T), \quad (6)$$

where  $\varepsilon_{vac} = \frac{1}{4}\langle \theta_{\mu\mu} \rangle$  is the nonperturbative vacuum energy density at  $T = 0$  and

$$\langle \theta_{\mu\mu} \rangle = -\frac{b}{32\pi^2} \langle G^2 \rangle + \sum_{q=u,d} m_q \langle \bar{q}q \rangle \quad (7)$$

is the trace of the energy-momentum tensor. In Eq.(6)  $P_\pi$  is the massive pions pressure

$$P_\pi = -3T \int \frac{d^3p}{(2\pi)^3} \ln(1 - e^{-\sqrt{\mathbf{p}^2 + M_\pi^2}/T}). \quad (8)$$

The quark and gluon condensates are given by the equations

$$\langle \bar{q}q \rangle(T) = -\frac{\partial P_{eff}}{\partial m_q}, \quad (9)$$

$$\langle G^2 \rangle(T) = \hat{D} P_{eff}, \quad (10)$$

where the operator  $\hat{D}$  is defined by the relation (3)

$$\hat{D} = \frac{32\pi^2}{b} (4 - T \frac{\partial}{\partial T} - \sum_q m_q \frac{\partial}{\partial m_q}). \quad (11)$$

Consider the  $T = 0$  case. One can use the low energy theorem for the derivative of the gluon condensate with respect to the quark mass [11]

$$\frac{\partial}{\partial m_q} \langle G^2 \rangle = \int d^4x \langle G^2(0) \bar{q}q(x) \rangle = -\frac{96\pi^2}{b} \langle \bar{q}q \rangle + O(m_q), \quad (12)$$

where  $O(m_q)$  stands for the terms linear in light quark masses. Then one arrives at the following relation<sup>2</sup>

$$\frac{\partial \varepsilon_{vac}}{\partial m_q} = -\frac{b}{128\pi^2} \frac{\partial}{\partial m_q} \langle G^2 \rangle + \frac{1}{4} \langle \bar{q}q \rangle = \frac{3}{4} \langle \bar{q}q \rangle + \frac{1}{4} \langle \bar{q}q \rangle = \langle \bar{q}q \rangle. \quad (13)$$

Note that three fourths of the quark condensate stem from the gluon part of the nonperturbative vacuum energy density. Along the same lines one arrives at the following expression for the gluon condensate

$$-\hat{D}\varepsilon_{vac} = -\frac{32\pi^2}{b} (4 - \sum_q m_q \frac{\partial}{\partial m_q}) (-\frac{b_0}{128\pi^2} \langle G^2 \rangle + \frac{1}{4} \sum_q m_q \langle \bar{q}q \rangle) = \langle G^2 \rangle. \quad (14)$$

In order to get the dependence of the quark and gluon condensates upon  $T$  use is made of the Gell-Mann- Oakes-Renner (GMOR) relation ( $\Sigma = |\langle \bar{u}u \rangle| = |\langle \bar{d}d \rangle|$ )

$$F_\pi^2 M_\pi^2 = -\frac{1}{2} (m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle = (m_u + m_d) \Sigma \quad (15)$$

Then we can find the following relations

$$\frac{\partial}{\partial m_q} = \frac{\Sigma}{F_\pi^2} \frac{\partial}{\partial M_\pi^2}, \quad (16)$$

$$\sum_q m_q \frac{\partial}{\partial m_q} = (m_u + m_d) \frac{\Sigma}{F_\pi^2} \frac{\partial}{\partial M_\pi^2} = M_\pi^2 \frac{\partial}{\partial M_\pi^2}, \quad (17)$$

$$\hat{D} = \frac{32\pi^2}{b} (4 - T \frac{\partial}{\partial T} - M_\pi^2 \frac{\partial}{\partial M_\pi^2}). \quad (18)$$

The pressure of the massive relativistic pion gas has the form

$$\begin{aligned} P_\pi(T) &= -\frac{3T^4}{2\pi^2} \int_0^\infty x^2 dx \ln(1 - e^{-\omega_\pi(x)}) = \frac{3T^4}{2\pi^2} \sum_{n=1}^\infty \frac{1}{n} \int_0^\infty x^2 dx e^{-n\omega_\pi(x)} \\ &= \frac{3M_\pi^2 T^2}{2\pi^2} \sum_{n=1}^\infty \frac{1}{n^2} K_2(n \frac{M_\pi}{T}), \end{aligned} \quad (19)$$

where  $\omega_\pi(x) = \sqrt{x^2 + M_\pi^2/T^2}$  and  $K_2$  is the Macdonald function. Making use of (9,13,16) and (19) one gets for the quark condensate

$$\frac{\Sigma(T)}{\Sigma} = 1 - \frac{3T^2}{4\pi^2 F_\pi^2} \int_0^\infty \frac{x^2 dx}{\omega_\pi(x)(e^{\omega_\pi(x)} - 1)} = 1 - \frac{3M_\pi T}{4\pi^2 F_\pi^2} \sum_{n=1}^\infty \frac{1}{n} K_1(n \frac{M_\pi}{T}). \quad (20)$$

In the chiral limit,  $M_\pi \ll T < T_c$ , the shift of the quark condensate is given by the standard temperature one-loop ChPT contribution to  $\langle \bar{q}q \rangle(T)$

---

<sup>2</sup>The equation (13) contains corrections  $\sim m_q \partial \langle \bar{q}q \rangle / \partial m_q \sim O(m_q)$  which are negligible for  $u$  and  $d$  quarks.

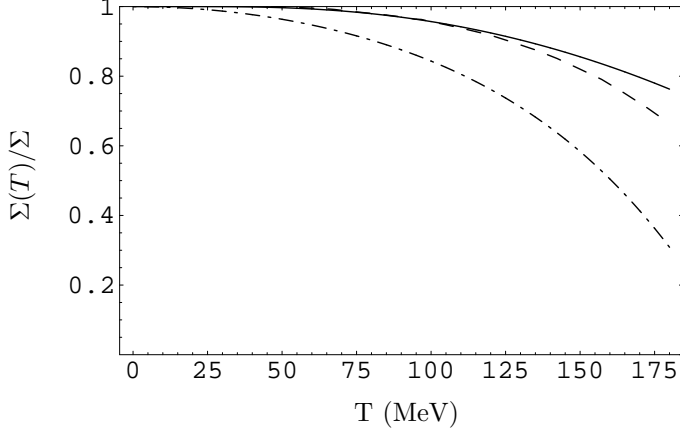


Figure 2: The quark condensate  $\Sigma(T)/\Sigma$  as a function of the temperature. The solid line corresponds to Eq.(20). The dash-dotted line corresponds to the three-loop ChPT formula (1) in chiral limit for  $N_f = 2$ . The dashed line is the numerical three-loop result of ChPT with non-zero quark mass (  $M_\pi=140$  MeV ) from Ref.[7].

$$\frac{\Delta\Sigma}{\Sigma} = -\frac{3T^2}{4\pi^2 F_\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} = -\frac{T^2}{8F_\pi^2}. \quad (21)$$

In the opposite limit of low temperatures,  $T \ll M_\pi$ , one obtains the following result

$$\frac{\Delta\Sigma}{\Sigma} = -\frac{3M_\pi T}{4\pi^2 F_\pi^2} K_1\left(\frac{M_\pi}{T}\right) \rightarrow -\frac{3M_\pi^{1/2} T^{3/2}}{2^{5/2} \pi^{3/2} F_\pi^2} e^{-M_\pi/T}. \quad (22)$$

The value of the quark condensate as a function of the temperature is depicted in Fig. 2. One can see that the gas formula (20) for the relativistic massive pions perfectly agrees with the numerical calculation of  $\Sigma(T)$  [7] within the framework of ChPT with non-zero quark mass. At the temperature  $T = M_\pi$  the deviation is only 2.5%.

4. Now let us consider the temperature dependence of the gluon condensate within the framework of the present approach. Eqs. (10), (14) and (18) yield

$$\frac{\langle G^2 \rangle(T)}{\langle G^2 \rangle} = 1 - \frac{24}{b} \frac{M_\pi^2 T^2}{\langle G^2 \rangle} \int_0^\infty \frac{x^2 dx}{\omega_\pi(x)(e^{\omega_\pi(x)} - 1)} = 1 - \frac{24}{b} \frac{M_\pi^3 T}{\langle G^2 \rangle} \sum_{n=1}^{\infty} \frac{1}{n} K_1\left(n \frac{M_\pi}{T}\right). \quad (23)$$

For  $M_\pi \ll T$  one gets

$$\frac{\Delta\langle G^2 \rangle}{\langle G^2 \rangle} = -\frac{24}{b} \frac{M_\pi^2 T^2}{\langle G^2 \rangle} \sum_{n=1}^{\infty} \frac{1}{n^2} = -\frac{4\pi^2}{b} \frac{M_\pi^2 T^2}{\langle G^2 \rangle}. \quad (24)$$

In chiral limit,  $M_\pi = 0$ , we have  $\Delta\langle G^2 \rangle(T) = 0$ , in agreement with the fact that a free gas of massless particles is conformally invariant. In the opposite nonrelativistic (low temperature) limit,  $T \ll M_\pi$ , one obtains

$$\frac{\Delta\langle G^2 \rangle}{\langle G^2 \rangle} = -\frac{24}{b} \frac{M_\pi^3 T}{\langle G^2 \rangle} K_1\left(\frac{M_\pi}{T}\right) \rightarrow -\frac{3 \cdot 2^{5/2} \pi^{1/2}}{b} \frac{M_\pi^{5/2} T^{3/2}}{\langle G^2 \rangle} e^{-M_\pi/T}. \quad (25)$$

On the other hand, as stated earlier, within ChPT and for zero quark mass the dependence of the gluon condensate on the temperature arises only at the three-loop level due to the interaction of the massless pions [9]. In Refs. [6, 7] the pressure of the  $N_f$  massless quarks has been calculated at the three-loop level with the result

$$P_{ChPT}^{N_f=2} = \frac{\pi^2}{30} T^4 \left\{ 1 + 4 \left( \frac{T^2}{12F_\pi^2} \right)^2 \ln \frac{\Lambda_p}{T} \right\} + O(T^{10}), \quad (26)$$

where the numerical value, relevant for the limit  $m_u = m_d = 0$  at fixed  $m_s$  is  $\Lambda_p \simeq 275$  MeV [7]. Then using (10), (11) and (16) one gets for the gluon condensate the result of Leutwyler [9]<sup>3</sup>

$$\frac{\langle G^2 \rangle(T)_{ChPT}}{\langle G^2 \rangle} = 1 - \frac{16\pi^4}{135b} \frac{T^8}{F_\pi^4 \langle G^2 \rangle} \left( \ln \frac{\Lambda_p}{T} - \frac{1}{4} \right). \quad (27)$$

In Fig.3 we show the temperature dependence of the gluon condensate at  $\langle G^2 \rangle = 0.5 \text{ GeV}^4$ .

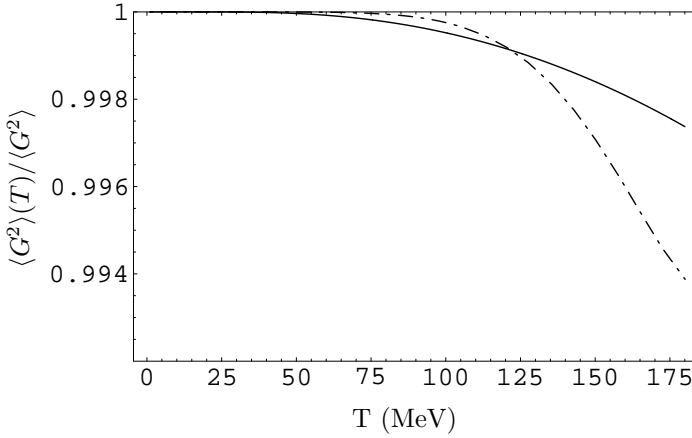


Figure 3: The gluon condensate  $\langle G^2 \rangle(T)/\langle G^2 \rangle$  as a function of the temperature. The solid line corresponds to Eq.(23) at  $\langle G^2 \rangle = 0.5 \text{ GeV}^4$ . The dash-dotted line corresponds to the three-loop result of ChPT in the chiral limit (Eq.(27)).

One can get an insight into the physical nature of small  $\langle G^2 \rangle$  shift with the increase of the temperature. The smallness of this quantity is due to the large value of  $\langle G^2 \rangle$  at  $T = 0$  as compared to the typical parameters of the thermal hadronic phase,  $\Delta\langle G^2 \rangle/\langle G^2 \rangle \propto M_\pi^2 T_c^2/\langle G^2 \rangle \simeq 10^{-3}$  at  $M_\pi = 0.14 \text{ GeV}$ ,  $T_c = 0.17 \text{ GeV}$  and  $\langle G^2 \rangle = 0.5 \text{ GeV}^4$ .

5. Within the above framework one can derive the thermodynamic relation for the quantum anomaly in the trace of the energy-momentum tensor of QCD. At low temperature the main contribution to the pressure comes from thermal excitations of massive pions. The general expression for the pressure reads

$$P_\pi = T^4 \varphi(M_\pi/T), \quad (28)$$

<sup>3</sup>In case  $M_\pi = 0$  one has  $(T\partial/\partial T - 4)P = \varepsilon - 3P = \langle \theta_{\mu\mu}^g \rangle_T$  [9].

<sup>4</sup>This value corresponds to the standard magnitude  $\langle \frac{\alpha_s}{\pi} F_{\mu\nu}^2 \rangle = 0.012 \text{ GeV}^4$  [22].

where  $\varphi$  is a function of the ratio  $M_\pi/T$  and in case of the dilute gas it is given by equation (19). Then the following relation is valid

$$(4 - T \frac{\partial}{\partial T} - M_\pi^2 \frac{\partial}{\partial M_\pi^2}) P_\pi = M_\pi^2 \frac{\partial P_\pi}{\partial M_\pi^2}. \quad (29)$$

Making use of (9,10), (13,22) and (29) one gets

$$\Delta \langle \bar{q}q \rangle = -\frac{\partial P_\pi}{\partial m_q}, \quad \Delta \langle G^2 \rangle = \frac{32\pi^2}{b} M_\pi^2 \frac{\partial P_\pi}{\partial M_\pi^2}, \quad (30)$$

where  $\Delta \langle \bar{q}q \rangle = \langle \bar{q}q \rangle_T - \langle \bar{q}q \rangle$  and  $\Delta \langle G^2 \rangle = \langle G^2 \rangle_T - \langle G^2 \rangle$ . In view of (17) one can recast (30) in the form

$$\Delta \langle G^2 \rangle = -\frac{32\pi^2}{b} \sum_q m_q \Delta \langle \bar{q}q \rangle \quad (31)$$

Let us divide both sides of (31) by  $\Delta T$  and take the limit  $\Delta T \rightarrow 0$ . This yields

$$\frac{\partial \langle G^2 \rangle}{\partial T} = -\frac{32\pi^2}{b} \sum_q m_q \frac{\partial \langle \bar{q}q \rangle}{\partial T}. \quad (32)$$

This can be rewritten as

$$\frac{\partial \langle \theta_{\mu\mu}^q \rangle}{\partial T} = \frac{\partial \langle \theta_{\mu\mu}^g \rangle}{\partial T} \quad (33)$$

where  $\langle \theta_{\mu\mu}^q \rangle = \sum m_q \langle \bar{q}q \rangle$  and  $\langle \theta_{\mu\mu}^g \rangle = (\beta(\alpha_s)/16\pi\alpha_s^2) \langle G^2 \rangle$  are correspondingly the quark and gluon contributions to the trace of the energy-momentum tensor. Note that when deriving this result use was made of the low energy GMOR relation, and therefore the thermodynamic relation (32,33) is valid in the light quark theory. Then at low temperature  $T$  when the excitations of massive hadrons and interactions of pions can be neglected, equation (33) becomes a rigorous QCD theorem. This can be easily verified via direct calculation using (20), (23) and the GMOR relation.

6. The present paper is devoted to the thermodynamic properties of QCD nonperturbative vacuum with two flavors at low temperature outside of the scope of perturbation theory. It was shown that at temperatures  $T \lesssim M_\pi$  the relativistic massive pions can be treated within the dilute gas approximation. Analytic temperature dependence of the quark condensate perfectly agrees, in the low temperature region,  $T \lesssim M_\pi$ , with the numerical calculations of  $\langle \bar{q}q \rangle(T)$  obtained at the three-loop level of ChPT with non-zero quark mass [7]. The gluon condensate slightly varies with the increase of the temperature, i.e. the situation is similar to the chiral perturbation theory. It was shown that the temperature derivatives of the anomalous and normal (quark massive term) contributions to the trace of the energy-momentum tensor in QCD with light quarks are equal to each other in the low temperature region.

As it was mentioned above the pion plays an exceptional role in thermodynamics of QCD due to the fact that its mass is numerically close to the phase transition temperature while the masses of heavier hadrons are several times larger than  $T_c$ . This was the reason we did not consider the role of massive states in the low temperature phase. This question was discussed in detail in Ref.[7]. It was shown there that at low temperatures, the contribution to  $\langle \bar{q}q \rangle$  generated by the massive states is very small, less than 5% if  $T$  below 100 MeV. At  $T = 150$  MeV, this contribution is of the order of 10%. The influence of thermal excitations of massive hadrons on the properties of the gluon and quark condensates in the framework of the conformal-nonlinear  $\sigma$ - model was also studied in detail in [23].

I am grateful to V.A.Rubakov and Yu.A.Simonov for useful discussions and comments. The financial support of RFFI grant 00-02-17836 and INTAS grant CALL 2000 N 110 is gratefully acknowledged.



## References

- [1] S. Weinberg, *Physica* **A 96** (1979) 327.
- [2] J. Gasser and H. Leutwyler, *Ann. Phys.* **158** (1984) 142.
- [3] J. Gasser and H. Leutwyler, *Nucl. Phys.* **B250** (1985) 465.
- [4] P. Binétrui and M.K. Gaillard, *Phys. Rev.* **D32** (1985) 931.
- [5] J. Gasser and H. Leutwyler, *Phys. Lett.* **B184** (1987) 83; **B188** (1987) 477; H. Neuberger, *Phys. Rev. Lett.* **60** (1988) 889.
- [6] H. Leutwyler, *Nucl. Phys. B* (proc. Suppl.) **4** (1988) 248.
- [7] P. Gerber and H. Leutwyler, *Nucl. Phys.* **B321** (1989) 387.
- [8] N.O. Agasian, *JETP Lett.* **57** (1993) 208.
- [9] H. Leutwyler, Restoration of Chiral Symmetry, Lecture given at Workshop on Effective Field Theories, Dobogoko, Hungary, 1991; Bern Univ. -BUTP-91-43.
- [10] F. Karsch, E. Laermann and A. Peikert, *Nucl. Phys.* **B605** (2001) 579; F. Karsch, hep-ph/0103314.
- [11] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B191** (1981) 301; *Sov. J. Part. Nucl.* **13** (1982) 224; A.A. Migdal and M.A. Shifman, *Phys. Lett.* **B114** (1982) 445.
- [12] P.J. Ellis, J.I. Kapusta and H.-B. Tang, *Phys. Lett.* **B443** (1998) 63.
- [13] I.A. Shushpanov, P.J. Ellis and J.I. Kapusta, *Phys. Rev.* **C59** (1999) 2931.
- [14] N.O. Agasian and I.A. Shushpanov, *JETP Lett.* **70** (1999) 717; *Phys. Lett.* **B472** (2000) 143.
- [15] N.O. Agasian, *Phys. Lett.* **B488** (2000) 39; *Yad.Fiz.* **64** (2001) 608.
- [16] S.H. Lee and I. Zahed, nucl-th/0006060.
- [17] I.T. Drummond, R.R. Horgan, P.V. Landshoff and A. Rebhan, *Phys. Lett.* **B460** (1999) 197.
- [18] N.O. Agasian, hep-ph/0104193.
- [19] E.V. Shuryak, *Phys. Lett.* **B207** (1988) 345; H. Leutwyler and A. Smilga, *Nucl. Phys.* **B342** (1990) 302.
- [20] J.L. Goity and H. Leutwyler, *Phys. Lett.* **B228** (1989) 517.
- [21] H. Bebie, P. Gerber, J.L. Goity and H. Leutwyler, *Nucl.Phys.* **B378** (1992) 95.
- [22] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B147** (1979) 385.
- [23] N.O. Agasian, D. Ebert and E.-M. Ilgenfritz, *Nucl.Phys.* **A637** (1998) 135.